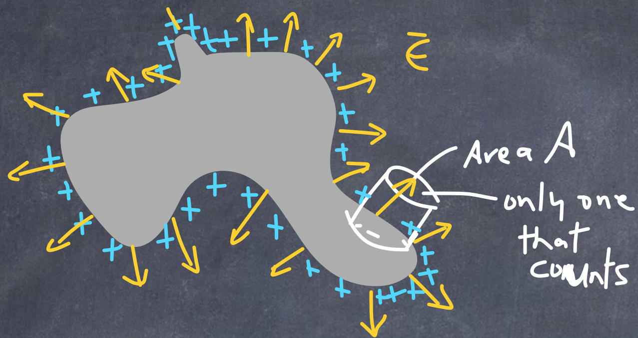
PHY117 HS2023

Week 9, Lecture 2
Nov. 15th, 2023
Prof. Ben Kilminster

$E = \frac{\sigma}{\varepsilon_0}$ close to a conductor



Near surface, $\bar{E} \parallel \hat{n}$

Also, for a conductor, charge is on surface,

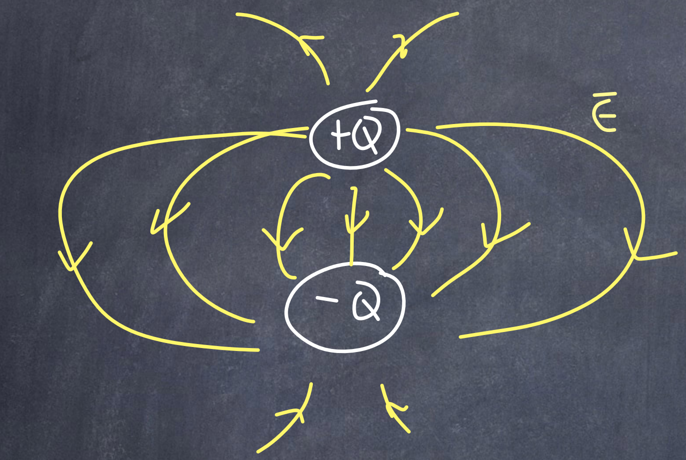
$$\oint_S \bar{E} \cdot \hat{n} \, dA = \frac{Q}{\varepsilon_0}$$

$$\sigma = \frac{Q}{A} \qquad Q = \sigma A$$

$$E \cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$$

$$\boxed{\bar{E} = \frac{\sigma}{\varepsilon_0} \hat{n} \quad \text{normal to surface}}$$

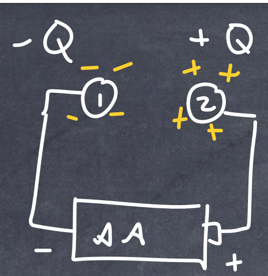
Capacitors :



Any two oppositely charged conductors form a capacitor

In a circuit diagram, the symbol for a capacitor is  or 

We can charge a capacitor by connecting each conductor to the opposite terminals of a battery.



$\Delta V = 1.5\,V$

Q on 1 is opposite to Q on 2.
There is a potential difference between the conductors, ΔV

we define the capacitance as $\dfrac{Q}{|\Delta V|} \equiv C$

The capacitance is the ability of 2 conductors to store electrical energy.

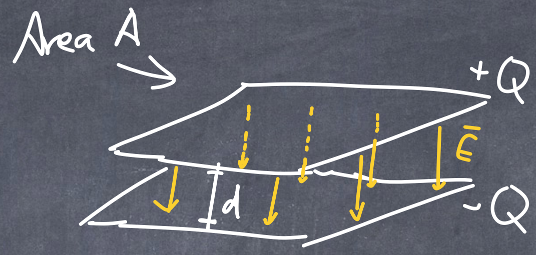
The units of capacitance are

$$1\ \text{Farad} = 1\,F = 1\,\frac{C}{V}$$

Energy stored in a capacitor:

$$U = \frac{1}{2} C (\Delta V)^2$$

what is the capacitance of a parallel-plate capacitor, with area A, separation d, and a charge $+Q$, $-Q$



we know that $E = \dfrac{\sigma}{\varepsilon_0}$, $\sigma = \dfrac{Q}{A}$ $\Rightarrow$ $E = \dfrac{Q}{\varepsilon_0 A}$ ①

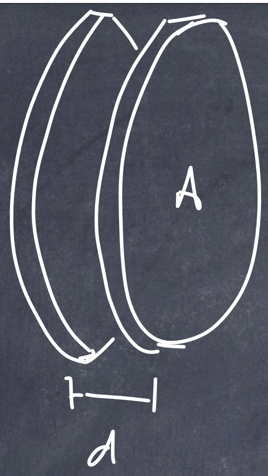
The field is uniform between the 2 plates.

$$|\Delta V| = \left| -\int_0^d \bar{E}\cdot d\bar{\ell} \right| = E\ell \Big]_0^d = Ed \overset{①}{=} \frac{Qd}{\varepsilon_0 A}$$

$$C = \frac{Q}{|\Delta V|} = \frac{\cancel{Q}}{\frac{\cancel{Q}\, d}{\varepsilon_0 A}} = \frac{\varepsilon_0 A}{d}$$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

capacitance only depends on the dimensions.



$\leftarrow$ capacitance is $C = \dfrac{\varepsilon_0 A}{d} = \dfrac{Q}{|\Delta V|}$

$$|\Delta V| = \frac{Q d}{\varepsilon_0 A}$$

If we put Q on our capacitor, and increase d, then ΔV increase

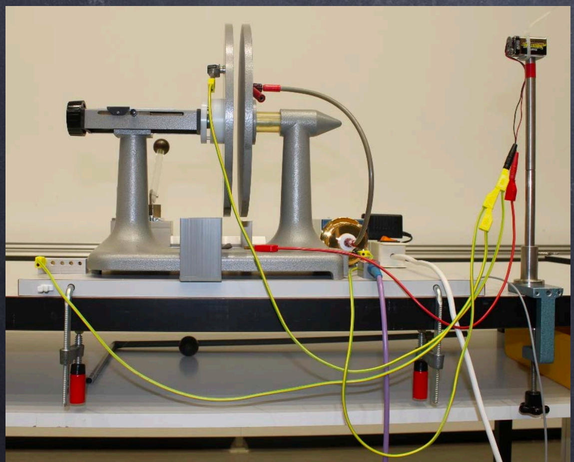
Energy stored in a capacitor

$$U = \tfrac{1}{2} C (\Delta V)^2$$

$$U = \frac{1}{2} \frac{\varepsilon_0 A}{\cancel{d}} E^2 d^{\cancel{2}}$$

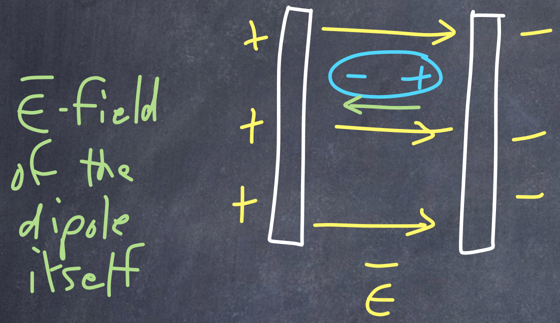
$$= \frac{1}{2} \varepsilon_0 A E^2 d$$

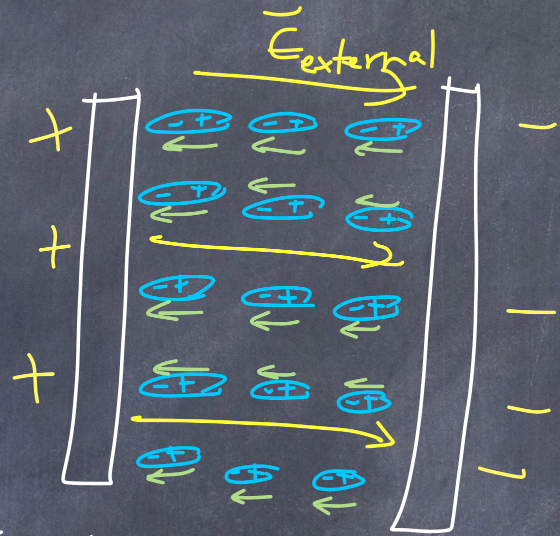
So we do work to increase the stored energy.



Dielectrics -

In a non-conducting material, we know that dipoles tend to align Opposite to the E-field





The $\bar{E}$-fields of the dipoles sum up

$$\text{total } \bar{E} \text{ inside} = \bar{E}_{external} - \bar{E}_{dipoles}$$

we call the non-conducting material a dielectric.

Each material has its own dielectric constant, K_d that weakens the electric field,

such that $E = \dfrac{E_0}{K_d}$

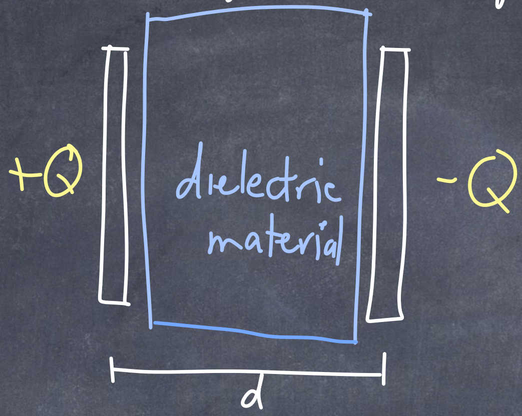
E_0 : E-field with no dielectric

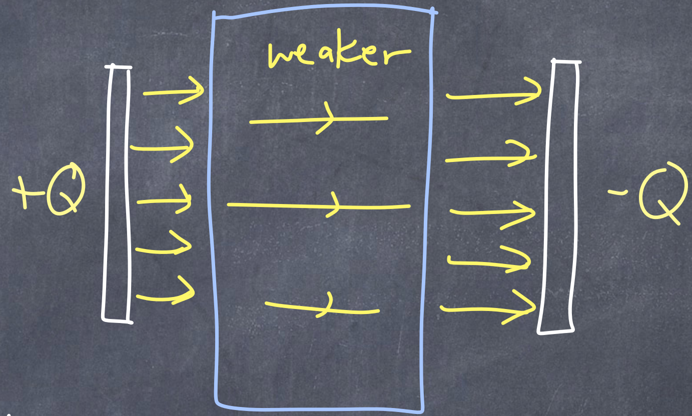
E : E-field with dielectric

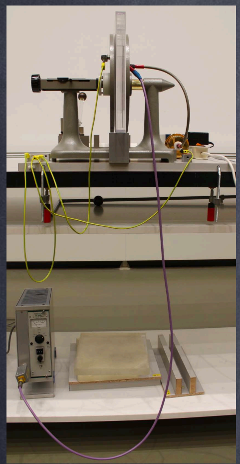
materials	K_d Dielectric constant (no units)
air	1.00059
water	80
paper	3.7
parafin	2
plexiglass	3.4

note: $K_d > 1$

Parallel-plate capacitor with a dielectric







Initially, we have ΔV_0 + $\mathcal{E}_0$ with no dielectric

$$|\Delta V_0| = \mathcal{E}_0 d$$

And we know that $\mathcal{E} = \dfrac{\mathcal{E}_0}{\kappa_d}$, so

we can get $\Delta V = \mathcal{E} d = \dfrac{\mathcal{E}_0 d}{\kappa_d} = \dfrac{|\Delta V_0|}{\kappa_d} \Rightarrow$ Adding dielectric decrease ΔV

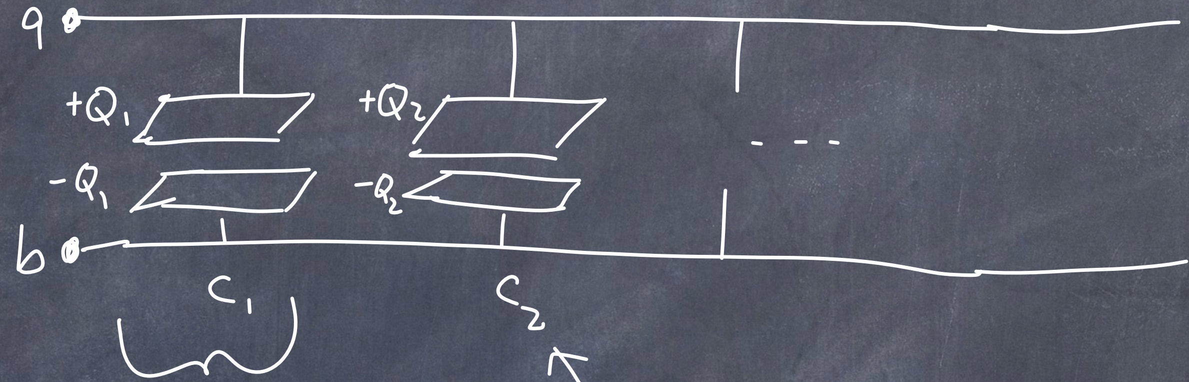
so $$C = \frac{Q}{|\Delta V|} = \frac{Q\,\kappa_d}{V_0} = \kappa_d C_0$$

For our parallel plate capacitor: $C = \dfrac{\kappa_d \mathcal{E}_0 A}{d}$

Combining capacitors:

when we combine capacitors in parallel, the capacitors are at the same potential

$V \equiv V_a - V_b$



$C_1 = \frac{Q_1}{V} \Rightarrow Q_1 = C_1 V \qquad Q_2 = C_2 V \qquad \ldots$

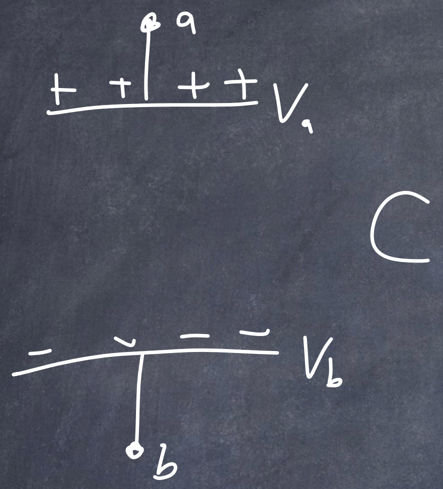
The total charge stored is $Q = Q_1 + Q_2 = C_1 V + C_2 V$

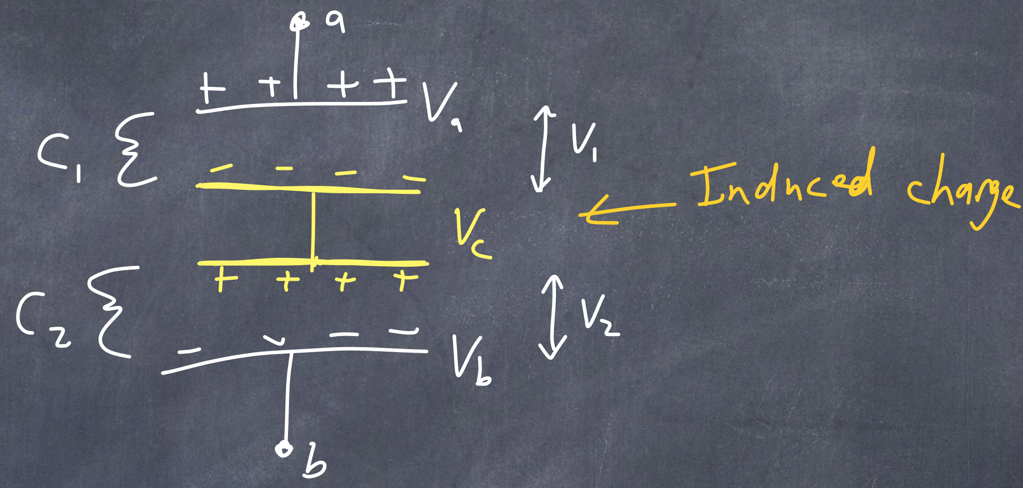
so $Q = (C_1 + C_2) V$

The equivalent capacitance of parallel capacitors is

$$\boxed{C_{eq} = C_1 + C_2 + \cdots = \frac{Q}{V}}$$

Combining capacitors in series:





$V \equiv V_a - V_b$ $\qquad$ $V_1 = V_a - V_c$ $\qquad$ $V_2 = V_c - V_b$

$V = \dfrac{Q}{C}$ $\qquad$ $V_1 = \dfrac{Q}{C_1}$ $\qquad$ $V_2 = \dfrac{Q}{C_2}$

The total potential difference $V = V_a - V_b$

so $$V = V_a - V_b = (V_a - V_c) + (V_c - V_b) = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$V = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \dots\right)$$

In general, $\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots}$

rule for adding capacitors in series.

If $C_1 = 2\mu F$ $C_2 = 2\mu F$

stronger → In parallel: $C_{eq} = C_1 + C_2 = 4\mu F$

weaker → In series: $\frac{1}{C_{eq}} = \frac{1}{2\mu F} + \frac{1}{2\mu F} \Rightarrow C_{eq} = 1\mu F$

Since $U = \frac{1}{2} C \Delta V^2 \Rightarrow$ energy is more when capacitors are in parallel

Capacitors in parallel

vs.

in series

more energy,
more spark.

Electrostatics: $E = 0$ in a conductor, V is the same everywhere in a conductor.

Electrodynamics: Current: rate of flow of electric charge.

$$\text{current} \rightarrow I = \frac{\Delta Q}{\Delta t} \quad \begin{array}{l} \leftarrow \text{charge} \\ \leftarrow \text{time} \end{array}$$

units $1\left[\frac{C}{s}\right] = 1\,A$

Amp = Ampere = A

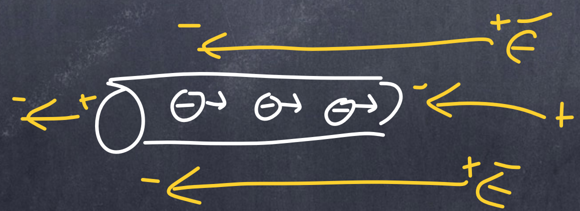
consider a conductor

$\bar{E} = 0$

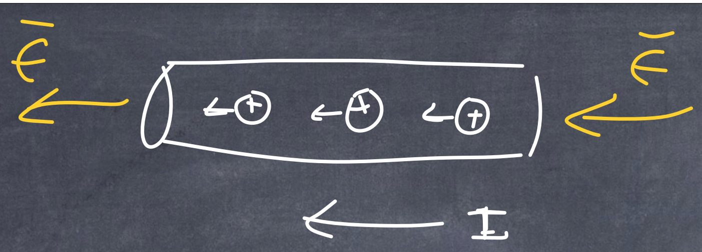
Free electrons in conductor move randomly due to thermal movement

$V \sim 10^6$ m/s

$\bar{E} \neq 0$

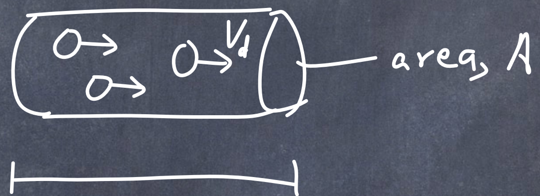


Electrons move in direction <u>opposite</u> to the E-field



By convention, the current I moves in the direction that positive charges would move.

$$n = \frac{\#\text{charge carriers}}{\text{volume}}$$



distance, $x = V_d \Delta t$

V_d : drift velocity

$\text{volume} = A \cdot x = A V_d \Delta t$

Total charge in this cylinder is:

$$\Delta Q = \frac{\#\text{charge carriers}}{\text{volume}} \cdot \text{volume} \cdot \text{charge per carrier}$$

$$= n \cdot \text{volume} \cdot e$$

$$\Delta Q = n A V_d \Delta t e$$

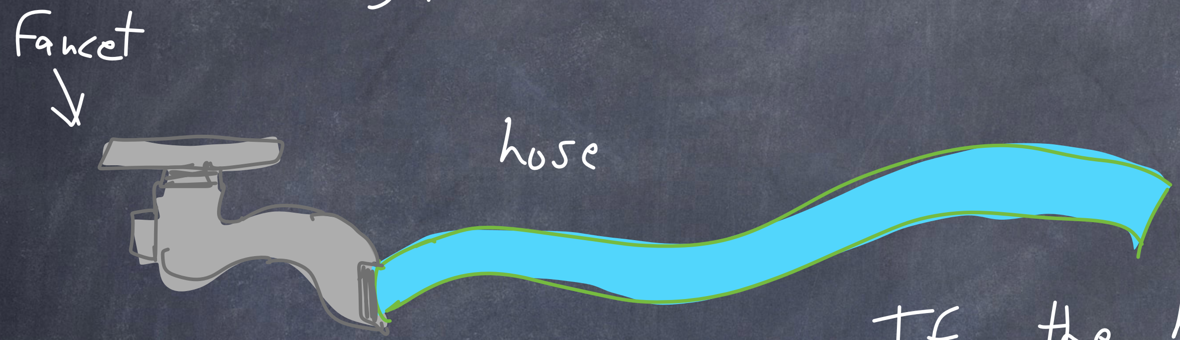
so $$I = \frac{\Delta Q}{\Delta t} = \frac{n A V_d \cancel{\Delta t} e}{\cancel{\Delta t}}$$

(exercise)

$$V_d = \frac{I}{nAe} \sim 10^{-5} \frac{m}{s}$$

If V_d is so slow, why does electricity seem so fast?

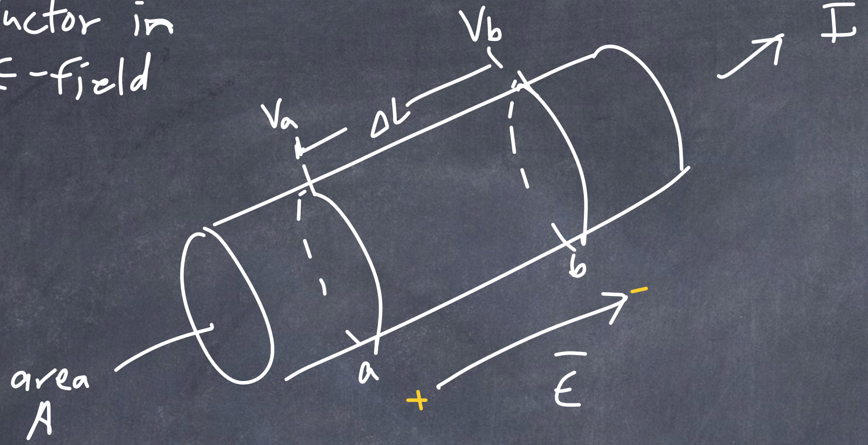
Analogy to a hose with water in it.



If the hose is already filled with water, then when you turn it on, so the fluid is incompressible, so the flow is instantaneous.

Electricity is $\sim$ instantaneous despite the slow electrons.

Conductor in an E-field



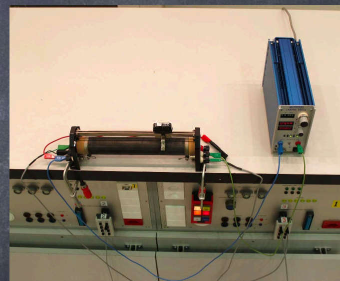
we know that

$$V = V_a - V_b = E\Delta L$$

$$V_a - V_b > 0$$

For most materials, the current is proportional to the potential difference:

$$\boxed{I \propto V}$$



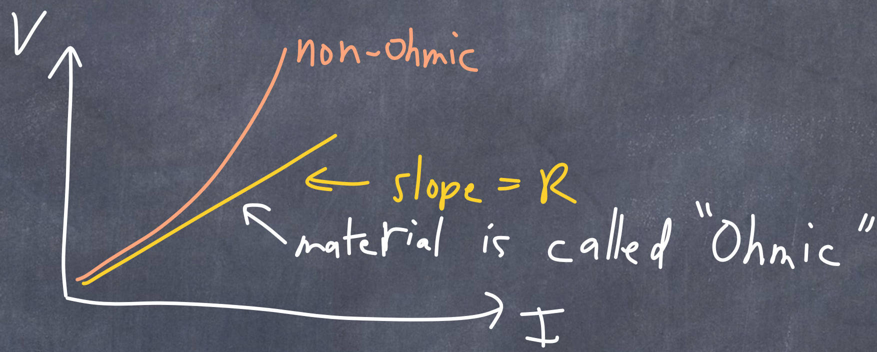
This constant of proportionality is $\frac{1}{R}$ where R is the resistance of the material

$$I = \left(\frac{1}{R}\right)V \Longrightarrow \boxed{V = IR}$$

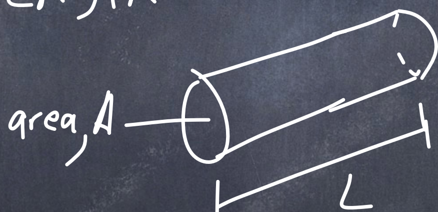
R has units of Ω (ohms)

where $1\,\Omega = 1\,\frac{V}{A} = \frac{1 \text{ volt}}{\text{Amp}}$

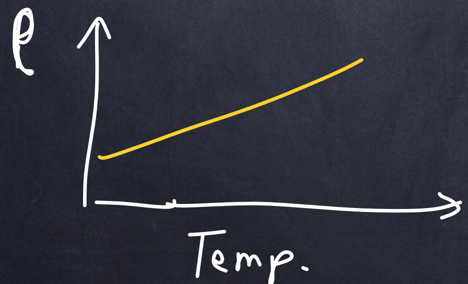
$V = IR$



The resistance R depends on the material, area, length



$R = \frac{L}{A}\rho$ ← "rho" is the resistivity



per material

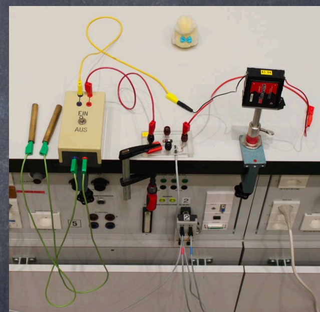
$$\underset{\text{resistivity}}{\rho} = \underset{\text{at } 20^\circ\text{C}}{\rho_{20}} \left[1 + \alpha \left(t_c - 20^\circ \right) \right]$$

Temp. in Celsius

material	ρ_{20} $[\Omega \cdot m]$	α $\left[\frac{1}{°C}\right]$
copper	1.7 E-8	3.9 E -3
aluminum	2.8 E-8	3.9 E -3
wood	$10^{8} - 10^{14}$	
glass	$10^{10} - 10^{14}$	

If R is small, then $I = \frac{V}{R}$ is big

more current in a copper tube than in a wooden one.



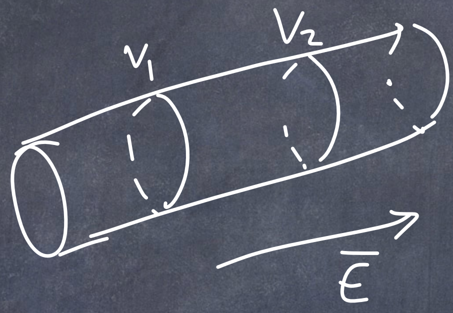
$V = 4.5\,V$

$I = 0.4\,mA = 0.0004\,A$

Sometimes people use

$$\text{Conductivity} = \frac{1}{\text{resistivity}}$$

Energy is lost in a conductor as electrical energy is converted into thermal energy.



Potential decrease from V_1 to V_2

$$V = V_1 - V_2$$

Loss in energy $\Delta U = \Delta Q (V_2 - V_1)$

ΔQ is the amount of charge flowing from V_1 to V_2

In time Δt, $\underbrace{\frac{\Delta U}{\Delta t}}_{\substack{\text{power} \\ \text{[Watts]}}} = \underbrace{\frac{\Delta Q}{\Delta t}}_{\substack{I \\ \text{[Amps]}}} \underbrace{(V)}_{\text{[volts]}}$

V is the potential difference here

1 Watt = 1 Amp · Volt = 1 A·V

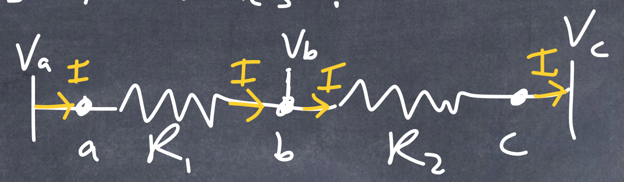
$$\boxed{\underset{\text{power}}{P} = IV}$$

since we know $V = IR$

$$\boxed{P = I^2 R = \frac{V^2}{R}}$$

The energy loss depends on R.
At constant I, a higher R produces more heat generated.
At constant V, a higher R produces less heat.

Resistors in series:



Equivalent resistance $\quad R_{eq} = R_1 + R_2 + \cdots$

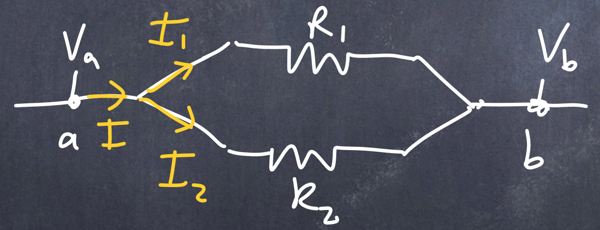
$$V_b = V_a - IR_1$$

$$V_c = V_a - IR_1 - IR_2$$

$$I_a = I_b = I_c = I$$

potential decreases, but the current stays the same.

Resistors in parallel:



$$I = I_1 + I_2$$

Equivalent resistance decreases (more ways for the current to flow)

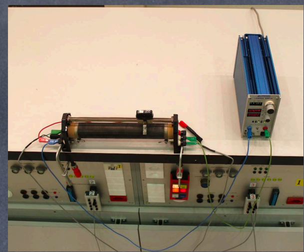
$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots}$$

voltage drop $V_a - V_b$ is the same across both pathways

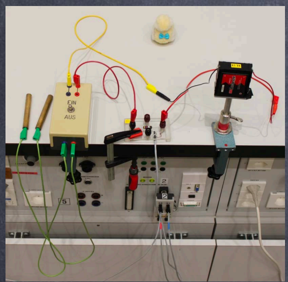
$$V_{ab} = V_a - V_b = I_1 R_1 = I_2 R_2$$

ES62

ES28



ES61



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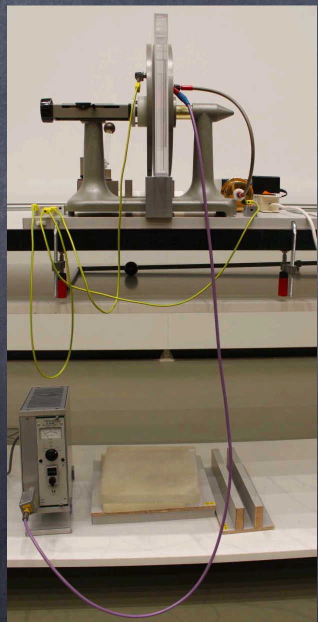

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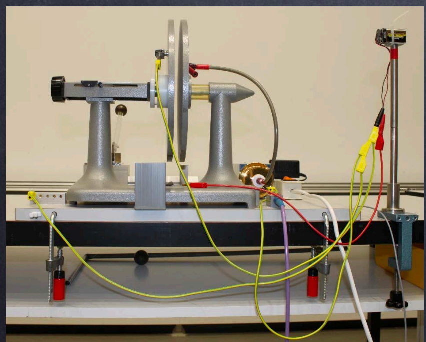

ES34